Inertial Navigation System Tests having Improved Observability of Error Sources

William S. Widnall,* Peter A. Grundy,†
Intermetrics, Inc., Cambridge, Mass.
and
Walter G. Murch‡
SAMSO, Los Angeles Air Force Station, Cal.

Optimal filtering of simulated inertial navigation system (INS) test data is used to evaluate alternate laboratory and flight test echniques, which are intended to determine the value of each significant source of navigation error. Tests of both gimbaled and strapdown systems are evaluated. The major problem preventing more accurate determination of the dozens of sources of error in an INS is the high correlation between the contributions of many of the sources of error. Laboratory test sequences and flight test trajectories are presented that reduce these correlations and improve the observability of the individual sources of error. Parametric studies include the effects of flight duration and distance, multi directional flights versus straight out-and-out flights, frequency and direction of maneuvers, and supersonic flights vs subsonic flights. The effects of the range instrumentation (reference system) accuracy and measurement frequency are demonstrated.

Introduction

THE performance of an inertial navigation system (INS) is a function of a large number of error sources, some of which are environmental (such as gravity deflections) and some of which are related to the instruments used in the INS mechanization. The objectives of testing an INS usually include the demonstration of its basic navigation accuracy. In addition, the test objectives may include the estimation of the values of the individual sources of error. These value estimates can be used to identify out-of-specification component performance. Such component performance information is the basis for the calibration values included in INS software to improve navigation accuracy. It also identifies, to the INS designer, areas in which component improvement efforts are needed.

A general difficulty in attempting to determine these performance parameters lies in the correlation of the contribution of many of the sources of error. Laboratory and flight test procedures can be designed, however, which reduce these correlations, thereby improving the observability of the individual sources of error.

Downs[1] by applying regression analysis to simulated INS flight data, has obtained some results on the effect of flight paths on the observability of error sources. In this paper, a high-order Kalman[2] filter is used as the basic tool with which to judge the suitability of the various test procedures in improving the observability of error sources. To avoid potential numerical problems, a square root filter formulation is used.[3,4] The unified INS error model approach of Britting[5] is used, however, with the error equations converted to first-order (state space) form, as in Ref. 6. Smoothing, as suggested by Nash et al.[7] is not used here because most of the gyro and accelerometer sources of error have been modeled as random constants. Fraser[8] has pointed out that smoothing cannot improve the estimates of constants with respect to the final estimates obtained from filtering. Additional details on the simulation techniques used are presented in the Appendix, and full discussion of the approach and the results obtained are presented in Ref. 9.

Measurement inputs to the filter consist only of position and velocity information; no precision attitude information is used. Reference sources considered for this study include a high-quality position and velocity reference, the parked aircraft known velocity, and the laboratory known position and velocity. The error models for each reference are given in Table 1. The high-quality position and velocity reference could be, for example, the Holloman Air Force Base CIRIS,[10] which obtains such accuracies or better by the optimal smoothing of reference inertial and precision ranging data.[11]

The testing of both a gimbaled and a strapdown INS is considered. The gimbaled INS is of high accuracy and has a local level wander-azimuth mechanization. The inertial measurement unit has three single-axis accelerometers, which permit mechanization of a complete baro-inertial altitude channel, and has two two-degree of freedom gyroscopes. The errors included in the filter model for this system are listed in Table 2. The assumed error source statistics are given in Table 3.

Some of the error sources are unobservable, and can be eliminated from the state. Given that no precision attitude references are to be used, three degrees of freedom of the gyro and accelerometer set as a whole are not separable from the

Table 1 Reference system accuracies

Error Component	1σ Random Error in Reference		
	Position and Velocity Ref.	Known Parked Aircraft Vel.	Known Lab Position and Vel.
East Position	20 ft	-	10 ft
North Position	20 ft	-	10 ft
Altitude	40 ft	-	10 ft
East Velocity	0.2 ft/sec	0.1 ft/sec	0.1 ft/sec
North Velocity	0.2 ft/sec	0.1 ft/sec	0.1 ft/sec
Vertical Velocity	0.4 ft/sec	0.1 ft/sec	0.1 ft/sec

Submitted July 3, 1974; presented as Paper 74-867 at the AIAA Mechanics and Control of Flight Conference, Anaheim, California, August 5-9, 1974; revision received March 20, 1975. Project supported by Air Force Special Weapons Center, 6585th Test Group, Air Force Systems Command, Holloman Air Force Base, N. M. The contributions of F. Rooney as initial technical monitor of these efforts are hereby acknowledged.

Index category: Navigation, Control, and Guidance Theory.

*Manager, Navigation, Guidance, and Control Department. Associate Fellow AIAA.

†Program Manager. Member AIAA.

‡Formerly Chief, Navigation Analysis Section, CIGTF, Holloman Air Force Base, N.M.

Table 2 Error sources of the local level INS

Position, Velocity, and Attitude Errors		
1.	$\delta\lambda$	Error in east longitude
2.	δL	Error in north latitude
3.	δh	Error in altitude
4.	δv_e	Error in east velocity
5.	δv_n	Error in north velocity
6.	δv_z	Error in vertical velocity
7.	ε_e	Attitude error east component
8.	ε_n	Attitude error north component
9.	ε_z	Attitude error up component
10.	$\delta\hat{a}$	Vertical acceleration error variable in altitude channel
G-Insensitive Gyro Drift		
11.	DX_f	X gyro drift rate
12.	DY_f	Y gyro drift rate
13.	DZ_f	Z gyro drift rate
G-Sensitive Gyro Drift Coefficients		
14.	DX_x	X gyro input axis g-sensitivity
15.	DX_y	Y gyro spin axis g-sensitivity
16.	DY_x	Y gyro spin axis g-sensitivity
17.	DY_y	Y gyro input axis g-sensitivity
18.	DZ_y	Z gyro spin axis g-sensitivity
19.	DZ_z	Z gyro input axis g-sensitivity
G^2-Sensitive Gyro Drift Coefficients		
20.	DX_{xy}	X gyro spin-input g^2 sensitivity
21.	DY_{xy}	Y gyro spin-input g^2 sensitivity
22.	DZ_{yz}	Z gyro spin-input g^2 sensitivity
Gyro Scale Factor Errors		
23.	GSF_x	X gyro scale factor error
24.	GSF_y	Y gyro scale factor error
25.	GSF_z	Z gyro scale factor error
Gyro Input Axis Misalignments		
26.	XG_y	X gyro input axis misalignment about Y
27.	XG_z	X gyro input axis misalignment about Z
28.	YG_x	Y gyro input axis misalignment about X
29.	YG_z	Y gyro input axis misalignment about Z
30.	ZG_x	Z gyro input axis misalignment about X
31.	ZG_y	Z gyro input axis misalignment about Y
Accelerometer Biases		
32.	AB_x	X accelerometer bias
33.	AB_y	Y accelerometer bias
34.	AB_z	Z accelerometer bias
Accelerometer Scale Factor Errors		
35.	ASF_x	X accelerometer scale factor error
36.	ASF_y	Y accelerometer scale factor error
37.	ASF_z	Z accelerometer scale factor error
Accelerometer Input Axis Misalignments		
38.	XA_y	X accelerometer input axis misalignment about Y
39.	XA_z	X accelerometer input axis misalignment about Z
40.	YA_x	Y accelerometer input axis misalignment about X
41.	YA_z	Y accelerometer input axis misalignment about Z
42.	ZA_x	Z accelerometer input axis misalignment about X
43.	ZA_y	Z accelerometer input axis misalignment about Y
Barometric Altimeter Errors		
44.	e_{po}	Error due to variation in altitude of a constant pressure surface
45.	e_{hsf}	Scale factor error
Gravity Deflections and Anomaly		
46.	δg_e	East deflection of gravity
47.	δg_n	North deflection of gravity
48.	δg_z	Gravity anomaly

platform attitude error states. We shall, therefore, arbitrarily define the accelerometer misalignments XA_y, XA_z and YA_x, to be zero. The X and Y accelerometers then define the platform coordinate system, and all other misalignments are defined relative to it. Furthermore, if the performance of the INS in the altitude channel is not of interest, certain error sources affecting the altitude and vertical velocity errors, but which do not cause significant horizontal errors, can be eliminated. These include AB_z, the Z accelerometer bias, ASF_z, the Z accelerometer scale factor, ZA_x and ZA_y, the Z accelerometer input axis misalignments about the X and Y axes, and δg_z, the gravity anomaly. The state dimension is, therefore, reduced from 48 to 40.

The strapdown system is a medium-accuracy INS which uses three single-axis accelerometers and three single degree-of -freedom gyros. The vertical channel is baro-stabilized. The sources of error included in the filter model for the strapdown INS are listed in Table 4.

Similarly as above, the three accelerometer misalignments YA_x, YA_z, and ZA_y (the Y accelerometer is pointed toward the aircraft nose, and the Z accelerometer is toward the right wing) can be eliminated, reducing the state dimension from 50 to 47. Error source statistics are presented in Table 5.

Both laboratory and flight tests are considered for each INS under test. Laboratory testing offers various advantages over flight test procedures: gravity deflections and anomaly are constant, and thus induce no growing errors in the INS under test; a highly accurate position and velocity reference is available, namely the surveyed laboratory position and known zero velocity; it offers a more controlled environment. Flight tests, on the other hand, allow one to escape the constrained specific force environment of the lab, namely the one-g field, which causes many distinct error sources to contribute in identical fashion to the INS position and velocity errors.

The enhancement of error source observability is a central issue in the design of laboratory and flight tests. A formal test for observability of a system can be applied, involving the computation of the rank of a matrix composed of various combinations of the state transition and measurement geometry matrices. If the matrix has less than full rank, only a number of linear combinations of the state variables equal to the rank of matrix, are observable.

Theoretical observability, however, does not consider the issue of measurement noise. A more practical test of observability lies in the reduction of the covariance of the state between its initial and final values. A test will have provided practical observability when the final filter computed uncertainty is significantly smaller than the initial value and at least as small as the desired accuracy of the error source determination.

Lack of observability is due to one of two cases: either 1) the contribution of the error source to the measurement is not significant compared to the measurement noise, or, 2) the

Table 3 Statistics of errors of the local level INS

RANDOM WALKS

State Variable Number	Error Source	Initial 1σ Value	Noise Spectral Density
11,12	X,Y (level) gyro drift rates	.003°/hr	(.003°/hr)²/hr
13	Z (azimuth) gyro drift rate	.005°/hr	(.005°/hr)²/hr
32,33	X,Y (horizontal) accelerometer biases	50 μg	(10 μg)²/hr
34	Z (altitude) accelerometer bias	100 μg	(10 μg)²/hr

FIRST ORDER MARKOV PROCESSES

State Variable Number	Error Source	1σ Value	Correlation Distance
44	Barometric altimeter time-varying error	500 ft.	250 n. mile
46	East deflection of gravity	26 μg	10 n. mile
47	North deflection of gravity	17 μg	10 n. mile
48	Gravity anomaly	35 μg	60 n. mile

RANDOM CONSTANTS

State Variable Number	Error Source	1σ Value
14 to 19	G-sensitive gyro drift coefficients	0.3°/hr/g
20 to 22	G²-sensitive gyro drift coefficients	0.04°/hr/g²
23,24	X,Y gyro scale factor errors	300 ppm
25	Z gyro scale factor error	1,000 ppm
26 to 31	Gyro input axis misalignments	40 arc sec
35 to 37	Accelerometer scale factor errors	150 ppm
39,41	X,Y accelerometer input axis misalignment about Z	180 arc sec
38,40 42,43	Other accelerometer input axis misalignments	30 arc sec
45	Barometric altimeter scale factor error	.03

contributions of the error source (to the measured position and velocity errors) are correlated with the contributions of one or more other sources of error; hence, the error source cannot be uniquely determined. Thus, a successful test must not only excite an error source but must also excite it in a manner which can be distinguished from other error source contributions to the measurements.

Laboratory Tests

The test sequences assumed represent covers-on system tests; the system is aligned using its own leveling and gyro compassing algorithms, and is then switched to the navigate mode. No special test modes are assumed. While this constraint degrades the local level system error source recovery capability vs what could be achieved in a test mode where the platform is reoriented, modification of the INS software to provide such a platform reorientation capability was not feasible under the simulation ground rules. For the data reduction process, the inertial indicated velocity and position are recorded, and the known laboratory velocity and position are used as measurement inputs to the optimal estimator which computes estimates of the error sources.

Lab Test of Local Level System

The problem of error source correlation is most severe for lab testing of local level systems, since the instruments are held at a fixed orientation with respect to the gravity vector. A useful test is permitted, however, by the assumed wander angle INS mechanization. By rotating the case prior to alignment, any desired orientation of the horizontal platform axes can be obtained.

A four-heading test of the local level INS was simulated. The INS is first aligned at zero wander angle (X platform axis east, Y axis north) and operated in the navigate mode for 84 min while recording its indicated horizontal position and velocity. The INS is next aligned at a wander angle of 180° and data from 84 min in the navigate mode are recorded. The sequence is repeated at wander angles of 90° and 270°.

The data are then processed by the 40-state estimator. The measurement set utilized by the filter consists of east position, north position, east velocity and north velocity, with assumed statistics as given in Table 1. A measurement set is utilized every 3 min. The initial state vector is zero. The initial covariance matrix is diagonal, with large values for the gyro and accelerometer state variances.

After processing the first period of navigation data, the final covariance matrix is stored. Before processing the next period of data, the covariances of those states whose values could be altered by realignment are reinitialized. They include the position errors, velocity errors, platform attitude errors, and the vertical acceleration errors. In addition, to represent the case where a significant period of time may have passed between the period of data-taking, the altimeter error sources are also reinitialized. This process is repeated for each of the remaining data periods.

The results presented in Table 6 are based on the final estimation error covariance matrix. The figure also includes the actual error values used in the simulation, and the initial one-sigma uncertainty assumed by the filter for each gyro and accelerometer error source.

Of the 26 error sources presented, only the horizontal gyro scale factors GSF_x and GSF_y have been estimated to a level better than the actual error source value. In addition to the individual error sources, five linear combinations which might be observable in this test are presented. They correspond to the apparent gyro drift rates, and the Z gyro input axis misalignment relative to the local gravity vector. As seen, the filter is only successful at estimating the horizontal azimuth-independent drift rates (*LIN8* and *LIN9*). The remaining three linear combinations contribute to the azimuth angular velocity error and are not as strongly observable as those contributing to the north angular velocity error.

Lab Test of Strapdown System

For strapdown systems, one has complete control of the attitude of the inertial sensor assembly and, thus, many of the sources of error can be made observable.

A test sequence which makes almost all the error sources observable has been simulated. The strapdown INS is placed in a two-degree-of-freedom mount of a rotating test table. We shall call the two mount angles the roll and pitch angles. The INS is placed in the mount such that the Y axis is along the mount roll axis and at zero roll angle the Z axis is along the mount pitch axis. At zero roll and pitch angles the Y and Z axes are perpendicular to (and the X axis is along) the table axis of rotation. The table is tilted such that the rotation axis lies horizontal and north/south. A positive table rate is defined to be an angular velocity directed north.

The test sequence is summarized in Table 7. First, at zero roll and pitch angles, 10 min of data in the navigate mode are recorded at zero table rate. Data recorded throughout the test include the east, north, and up components of indicated velocity plus the indicated longitude, latitude, and altitude. Then, the table is rotated at 6°/sec for one min (a 360° rotation). Then, the table is stopped and 9 min are allowed to elapse to let attitude error propagate into measurable velocity and position error. During the 9-min period of zero table rate,

Table 4 Error sources of strapdown INS

No.	Symbol	Description
Position, Velocity, and Attitude Errors		
1.	$\delta\lambda$	Error in east longitude
2.	δL	Error in north latitude
3.	δh	Error in altitude
4.	δv_e	Error in east velocity
5.	δv_n	Error in north velocity
6.	δv_z	Error in vertical velocity
7.	ε_e	Attitude error east component
8.	ε_n	Attitude error north component
9.	ε_z	Attitude error up component
G-Insensitive Gyro Drift		
10.	DX_f	X gyro drift rate
11.	DY_f	Y gyro drift rate
12.	DZ_f	Z gyro drift rate
G-Sensitive Gyro Drift Coefficients		
13.	DX_i	X gyro input axis g-sensitivity
14.	DX_s	X gyro spin axis g-sensitivity
15.	DY_i	Y gyro input axis g-sensitivity
16.	DY_s	Y gyro spin axis g-sensitivity
17.	DZ_i	Z gyro input axis g-sensitivity
18.	DZ_s	Z gyro spin axis g-sensitivity
G^2-Sensitive Gyro Drift Coefficients		
19.	DX_{os}	X gyro output-spin g^2 sensitivity
20.	DY_{os}	Y gyro output-spin g^2 sensitivity
21.	DZ_{os}	Z gyro output-spin g^2 sensitivity
Gyro Scale Factor Errors		
22.	GSF_x^+	X gyro positive scale factor error
23.	GSF_x^-	X gyro negative scale factor error
24.	GSF_y^+	Y gyro positive scale factor error
25.	GSF_y^-	Y gyro negative scale factor error
26.	GSF_z^+	Z gyro positive scale factor error
27.	GSF_z^-	Z gyro negative scale factor error
Gyro Input Axis Misalignments		
28.	XG_y	X gyro input axis misalignment about Y
29.	XG_z	X gyro input axis misalignment about Z
30.	YG_x	Y gyro input axis misalignment about X
31.	YG_z	Y gyro input axis misalignment about Z
32.	ZG_x	Z gyro input axis misalignment about X
33.	ZG_y	Z gyro input axis misalignment about Y
Accelerometer Biases		
34.	AB_x	X accelerometer bias
35.	AB_y	Y accelerometer bias
36.	AB_z	Z accelerometer bias
Accelerometer Scale Factor Errors		
37.	ASF_x	X accelerometer scale factor error
38.	ASF_y	Y accelerometer scale factor error
39.	ASF_z	Z accelerometer scale factor error
Accelerometer Input Axis Misalignments		
40.	XA_y	X accelerometer input axis misalignment about Y
41.	XA_z	X accelerometer input axis misalignment about Z
42.	YA_x	Y accelerometer input axis misalignment about X
43.	YA_z	Y accelerometer input axis misalignment about Z
44.	ZA_x	Z accelerometer input axis misalignment about X
45.	ZA_y	Z accelerometer input axis misalignment about Y
Barometric Altimeter Errors		
46.	e_{po}	Error due to variation in altitude of a constant pressure surface
47.	e_{hsf}	Scale factor error
Gravity Deflections and Anomaly		
48.	δg_e	East deflection of gravity
49.	δg_n	North deflection of gravity
50.	δg_z	Gravity anomaly

the mount pitch angle is changed to 45°. The simulation accomplishes these pitch or roll changes at smooth rates, not exceeding 6°/sec. At the new pitch angle, a second 360° table rotation at 6°/sec is executed, followed by a 9-min rest. A third rotation takes place at 135° pitch angle. A fourth at 180° pitch angle. Finally, at 180° pitch angle a slow rotation is executed (0.1°/sec for 30 min, a 180° rotation). This completes the first third of the test, during which the Z axis was perpendicular to the table axis of rotation.

The second third is similar to the first third except that the X axis is maintained perpendicular to the table axis of rotation. The last third is similar but with the Y axis held perpendicular to the axis of rotation. The total test duration in the navigate mode is 3 hr, 46 min.

In addition to the velocity and position measurements, the post-test processor requires the orientation of the system with respect to east-north-up throughout the test. This may be obtained either from the transformation matrix computed by the strapdown system under test or from recorded table rotation angle and mount roll and pitch angles. High accuracy is not required of this attitude information as it is used only to compute elements in the state transition matrix.

The test data have been processed by the optimal filter. The results are presented in Table 8. Excellent estimates are obtained for the values of the six g-sensitive gyro drift coefficients and the six positive and negative gyro scale factor errors. The filter computed uncertainty is smaller than the quantity of interest for all sources of error except the accelerometer scale factors. These computed uncertainties indicate that all sources of error except the accelerometer scale factor errors may be accurately determined from this laboratory test.

Flight Tests

Flight tests allow generation of varied specific force and angular velocity inputs to the INS. Maneuvers excite specific force dependent error sources. Flight at different headings may excite angular velocity dependent error sources.

The reference measurements of aircraft position and velocity in flight correspond to a simulated high-accuracy reference, as described in Table 1. On the ground, measurements are the known zero velocity, as described in Table 1. One measurement set per 3 min is utilized by the optimal filter, both for on the ground and in flight portions.

Flight Test of Local Level INS

The first flight path explored is an L-shaped path containing north-south and east-west legs. The INS is aligned

Table 5 Statistics of errors of strapdown INS

RANDOM WALKS

State Variable Number	Error Source	Initial 1σ Value	Noise Spectral Density
10	X gyro drift rate (not maneuvering)	.025°/hr	(.03°/hr)^2/hr
11,12	Y,Z gyro drift rates (not maneuvering)	.018°/hr	(.02°/hr)^2/hr
34	X accelerometer bias	30 μg	(10 μg)^2/hr
35,36	Y,Z accelerometer bias	20 μg	(10 μg)^2/hr

FIRST ORDER MARKOV PROCESSES

State Variable Number	Error Source	1σ Value	Correlation Distance
46	Barometric altimeter time-varying error	500 ft.	250 n. mile
48	East deflection of gravity	26 μg	10 n. mile
49	North deflection of gravity	17 μg	10 n. mile
50	Gravity anomaly	35 μg	60 n. mile

RANDOM CONSTANTS

State Variable Number	Error Source	1σ Value
13 to 18	G-sensitive gyro drift coefficients	0.2°/hr/g
19 to 21	G^2-sensitive gyro drift coefficients	0.07°/hr/g^2
22 to 27	Gyro scale factor errors	70 ppm
28 to 33	Gyro input axis misalignments	10 arc sec
37 to 39	Accelerometer scale factor errors	35 ppm
40 to 45	Accelerometer input axis misalignments	10 arc sec
47	Barometric altimeter scale factor error	.03

Table 6 Four-heading laboratory test results

Source	Units	Sim. Value	Filter Comp. Uncertainty	Initial Uncertainty
DXF	DEG/HR	0.00300	0.05830	0.10004
DYF	DEG/HR	0.00300	0.05843	0.10004
DZF	DEG/HR	-0.30321	7.07141	10.00384
DXX	DEG/HR/G	0.300	7.215	10.001
DXY	DEG/HR/G	0.300	7.667	10.001
DYX	DEG/HR/G	0.300	7.312	10.001
DYY	DEG/HR/G	0.300	7.564	10.001
DZY	DEG/HR/G	0.300	9.967	10.001
DZX	DEG/HR/G	0.300	7.074	10.001
DXXY	DEG/HR/G2	0.0400	1.0001	1.0001
DYXY	DEG/HR/G2	0.0400	1.0001	1.0001
DZYZ	DEG/HR/G2	0.0400	1.0001	1.0001
GSFX	PPM	300.	105.	10000.
GSFY	PPM	300.	103.	10000.
GSFZ	PPM	1000.	10000.	10000.
XGY	ARC MIN	0.667	24.480	30.001
XGZ	ARC MIN	-0.667	27.356	30.001
YGX	ARC MIN	-0.667	24.171	30.001
YGZ	ARC MIN	0.667	27.405	30.001
ZGX	ARC MIN	0.667	4.795	30.001
ZGY	ARC MIN	-0.667	4.791	30.001
ABX	UG	50.	973.	1000.
ABY	UG	50.	973.	1000.
ASFX	PPM	150.	9787.	10000.
ASFY	PPM	150.	9809.	10000.
YAZ	ARC MIN	-3.001	29.519	30.001
Linear Comb.:		LIN3=DZF+DZZ*G+GSFZ*WIEZ		
LIN3	DEG/HR	0.00476	0.01548	14.15386
	LIN6=ZGX-ABY/G	LIN7=ZGY+ABX/G		
LIN6	ARC MIN	0.495	3.480	30.197
LIN7	ARC MIN	-0.495	3.475	30.197
	LIN8=DXF+XGY*WIEZ+ABX*WIEZ/GZ		LIN9=DYF-YGX*WIEZ+ABY*WIEZ/GZ	
LIN8	DEG/HR	0.00499	0.00226	0.12300
LIN9	DEG/HR	0.00499	0.00534	0.12300

with the X axis east and the Y axis north (zero wander angle). The aircraft holds on the runway for 21 min. Then it takes off and flies north while climbing. The aircraft simulated is a subsonic transport. Below 10,000 ft the aircraft speed is held at about 200 knots TAS (true air speed) and the rate of climb is 1000 ft/min. Above 10,000 ft the speed is 250 knots IAS (Indicated air speed) and the rate of climb is 2000 ft/min. The aircraft levels off at 20,000 ft and continues true north at 250 knots IAS (390 knots TAS). Twenty-one minutes after takeoff the aircraft initiates a 30°-banked 180° right turn and then flies south. Over the starting point, the aircraft turns left with a 30°-banked turn. It proceeds east for 23 min, then executes a 30°-banked 180° right turn and returns. It descends from 20,000 ft to 5500 ft during the end of the return leg and then lands. After stopping, an additional 19 min of navigation data is recorded for post flight processing.

Four L-shaped flight paths have been flown, differing only in the leg durations. The approximate leg durations on these flights are: 12 min, 21 min, 42 min, and 84 min. All four flights begin with 21 min on the ground before take-off and end with about 21 min on the ground after landing.

Table 7 Test sequence, strapdown laboratory test

MOUNT ROLL ANGLE (DEG)	MOUNT PITCH ANGLE (DEG)	TABLE ROTATION RATE (DEG/SEC)	TEST SEGMENT DURATION (MIN)
0	0	0.0	10
0	0	6.0	1
0	0	0.0	5
0	45	0.0	4
0	45	6.0	1
0	45	0.0	5
0	135	0.0	4
0	135	6.0	1
0	135	0.0	5
0	180	0.0	4
0	180	6.0	1
0	180	0.0	5
0	180	0.1	30
-90	270	0.0	4
-90	270	6.0	1
-90	270	0.0	5
-90	315	0.0	4
-90	315	6.0	1
-90	315	0.0	5
-90	405	0.0	4
-90	405	6.0	1
-90	405	0.0	5
-90	450	0.0	4
-90	450	6.0	1
-90	450	0.0	5
-90	450	0.1	30
-90	540	0.0	4
-90	540	6.0	1
-90	540	0.0	5
-45	540	0.0	4
-45	540	6.0	1
-45	540	0.0	5
45	540	0.0	4
45	540	6.0	1
45	540	0.0	5
90	540	0.0	4
90	540	6.0	1
90	540	0.0	5
90	540	0.1	30
90	540	0.0	10

Table 8 Strapdown laboratory tests results

Source	Units	Sim. Value	Filter Comp. Uncertainty	Initial Uncertainty
DXF	DEG/HR	0.02500	0.01385	1.00038
DYF	DEG/HR	0.01800	0.01511	1.00038
DZF	DEG/HR	0.01800	0.01014	1.00038
DXX	DEG/HR/G	0.200	0.024	10.001
DXZ	DEG/HR/G	0.200	0.007	10.001
DYY	DEG/HR/G	0.200	0.020	10.001
DYZ	DEG/HR/G	0.200	0.009	10.001
DZZ	DEG/HR/G	0.200	0.023	10.001
DZY	DEG/HR/G	0.200	0.005	10.001
DXYZ	DEG/HR/G2	0.0700	0.0115	1.0001
DYXZ	DEG/HR/G2	0.0700	0.0272	1.0001
DZXY	DEG/HR/G2	0.0700	0.0098	1.0001
GSFX+	PPM	70.	6.	10000.
GSFX-	PPM	70.	3.	10000.
GSFY+	PPM	70.	5.	10000.
GSFY-	PPM	70.	4.	10000.
GSFZ+	PPM	70.	4.	10000.
GSFZ-	PPM	70.	3.	10000.
XGY	ARC MIN	0.167	0.069	30.001
XGZ	ARC MIN	-0.167	0.059	30.001
YGX	ARC MIN	-0.167	0.049	30.001
YGZ	ARC MIN	0.167	0.056	30.001
ZGX	ARC MIN	0.167	0.050	30.001
ZGY	ARC MIN	-0.167	0.070	30.001
ABX	UG	30.	16.	1000.
ABY	UG	20.	17.	1000.
ABZ	UG	20.	15.	1000.
ASFX	PPM	35.	277.	10000.
ASFY	PPM	35.	277.	10000.
ASFZ	PPM	35.	278.	10000.
XAY	ARC MIN	-0.167	0.081	30.001
XAZ	ARC MIN	0.167	0.063	30.001
ZAX	ARC MIN	-0.167	0.055	30.001

Table 9 L-shaped paths of various durations

			LEG DURATIONS				
			12 min.	21 min.	42 min.	84 min.	
Source	Units	Sim. Value	Filter Comp Uncer	Filter Comp Uncer	Filter Comp Uncer	Filter Comp Uncer	Initial Uncer
DXF	DEG/HR	0.00300	0.05821	0.05465	0.03812	0.02169	0.10004
DYF	DEG/HR	0.00300	0.05862	0.05524	0.03942	0.02087	0.10004
DZF	DEG/HR	-0.30321	6.92237	6.82405	6.76284	6.70669	10.00384
DXX	DEG/HR/G	0.300	0.140	0.117	0.110	0.111	10.001
DXY	DEG/HR/G	0.300	0.159	0.133	0.133	0.133	10.001
DYX	DEG/HR/G	0.300	0.166	0.135	0.125	0.123	10.001
DYY	DEG/HR/G	0.300	0.180	0.152	0.153	0.159	10.001
DZY	DEG/HR/G	0.300	2.207	2.087	1.991	1.965	10.001
DZZ	DEG/HR/G	0.300	6.914	6.807	6.738	6.676	10.001
DXXY	DEG/HR/G2	0.0400	0.9196	0.8644	0.8326	0.8258	1.0001
DYXY	DEG/HR/G2	0.0400	0.9233	0.8880	0.8857	0.8823	1.0001
DZYZ	DEG/HR/G2	0.0400	0.9953	0.9953	0.9953	0.9953	1.0001
GSFX	PPM	300.	2005.	830.	463.	389.	10000.
GSFY	PPM	300.	1625.	735.	375.	251.	10000.
GSFZ	PPM	1000.	9990.	9902.	9830.	8386.	10000.
XGY	ARC MIN	0.667	24.641	23.036	16.469	9.510	30.001
XGZ	ARC MIN	-0.667	4.712	2.058	1.193	1.030	30.001
YGX	ARC MIN	-0.667	24.717	23.467	16.680	9.042	30.001
YGZ	ARC MIN	0.667	7.277	3.261	1.355	0.839	30.001
ZGX	ARC MIN	0.667	25.260	19.685	13.095	10.423	30.001
ZGY	ARC MIN	-0.667	27.177	19.864	11.473	6.786	30.001
ABX	UG	50.	956.	932.	938.	934.	1000.
ABY	UG	50.	949.	919.	929.	914.	1000.
ASFX	PPM	150.	185.	166.	156.	159.	10000.
ASFY	PPM	150.	175.	192.	163.	149.	10000.
YAZ	ARC MIN	-3.001	0.830	0.899	0.844	0.834	30.001

Table 10 Single axis flights compared with an L-shaped flight

			N-S 84 min Legs	E-W 84 min Legs	NE-SW 84 min Legs	N-S-E-W 42 min Legs
Source	Units	Sim. Value	Filter Comp Uncer	Filter Comp Uncer	Filter Comp Uncer	Filter Comp Uncer
DXF	DEG/HR	0.00300	0.07950	0.05840	0.06934	0.03912
DYF	DEG/HR	0.00300	0.08201	0.05834	0.07100	0.03942
DZF	DEG/HR	-0.30321	7.03082	6.76695	6.78721	6.76284
DXX	DEG/HR/G	0.300	1.531	0.128	0.583	0.110
DXY	DEG/HR/G	0.300	0.142	1.291	0.621	0.133
DYX	DEG/HR/G	0.300	1.629	0.151	0.698	0.125
DYY	DEG/HR/G	0.300	0.174	1.472	0.744	0.153
DZY	DEG/HR/G	0.300	3.404	4.896	3.106	1.991
DZZ	DEG/HR/G	0.300	7.035	6.741	6.790	6.738
DXXY	DEG/HR/G2	0.0400	0.8872	0.9355	0.7860	0.8326
DYXY	DEG/HR/G2	0.0400	0.9330	0.9601	0.8551	0.8857
DZYZ	DEG/HR/G2	0.0400	0.9957	0.9963	0.9956	0.9953
GSFX	PPM	300.	871.	5166.	3144.	463.
GSFY	PPM	300.	4654.	506.	3193.	375.
GSFZ	PPM	1000.	9886.	10000.	9926.	9830.
XGY	ARC MIN	0.667	13.520	24.486	16.536	16.469
XGZ	ARC MIN	-0.667	15.812	2.393	10.837	1.193
YGX	ARC MIN	-0.667	13.327	24.491	16.195	16.680
YGZ	ARC MIN	0.667	1.732	11.997	10.533	1.355
ZGX	ARC MIN	0.667	29.818	11.943	22.105	13.095
ZGY	ARC MIN	-0.667	9.723	29.637	22.231	11.473
ABX	UG	50.	934.	939.	932.	938.
ABY	UG	50.	919.	920.	909.	929.
ASFX	PPM	150.	431.	235.	316.	156.
ASFY	PPM	150.	248.	447.	278.	163.
YAZ	ARC MIN	-3.001	1.996	1.850	1.552	0.844
LINEAR COMB.: LIN1+DXF+XGY*WIEZ			LIN2=DYF-YGX*WIEZ		LIN3=DZF+DZZ*G+GSFX*WIEZ *G+GSFZ*WIEZ	
LIN1	DEG/HR	0.00458	0.05805	0.00958	0.04021	0.00874
LIN2	DEG/HR	0.00458	0.05942	0.00954	0.04090	0.00889
LIN3	DEG/HR	0.00476	0.10673	0.03736	0.07727	0.04491

The filtering results are presented in Table 9. Using the 21-min-leg flight as a baseline, it can be seen that only five error sources have been determined to an accuracy better than the magnitude of the simulated error sources. These are: the horizontal gyro horizontal g-sensitivities DX_x, DX_y, DY_x, and DY_y, and the nonorthogonality of the Y accelerometer input axis relative to the X accelerometer YA_z. The horizontal accelerometer scale factors ASF_x and ASF_y are determined to an accuracy comparable to the simulated values. The uncertainty in the horizontal gyro torquer scale factors GSF_x and GSF_y are almost three times the simulated values, which is not very helpful. The uncertainties in the estimates of all the other sources of error are significantly larger than the quantities of interest.

The acceleration sensitive sources of error are determined more accurately with the 21-min legs than with the 12-min legs. But increasing the leg durations to 42 or 84 min does not yield a further improvement. The acceleration sensitive sources of error include: the g-sensitive gyro drifts, the g^2-sensitive gyro drifts, the accelerometer scale factor errors, and the accelerometer input axis nonorthogonality.

Several groups of error sources are seen to be better estimated with longer duration legs. These include the g-insensitive gyro drifts, the gyro torquer scale factor errors, and the gyro input axis misalignments.

Note that there is nothing particular good or bad about 42-min-leg durations. The north 42-min leg, followed by south 42-min leg is a round trip having an 84-min period. The same is true of the east 42-min leg, followed by the west 42-min leg. Flight paths having a strong Fourier component at the 84-min Schuler period can induce large Schuler error oscillations in the INS. It might seem that inducing Schuler oscillations would significantly enhance the observability of many of the sources of error. On the other hand having several sources of error induce similar Schuler oscillations might imply that the sources of error are highly correlated and, therefore, difficult to separate. The simulation shows neither extreme to be the case. The progression of the results is quite uniform in passing from 21-min legs to 42-min legs to 84-min legs.

Single axis flights do not compare favorably with an L-shaped flight of the same duration. Table 10 presents comparisons between a north-south-east-west L-shaped flight,

Table 11 Random flight

Source	Units	Sim. Value	Filter Comp Uncertainty	Initial Uncertainty
DXF	DEG/HR	0.00300	0.05503	0.10004
DYF	DEG/HR	0.00300	0.05658	0.10004
DZF	DEG/HR	-0.30321	6.91089	10.00384
DXX	DEG/HR/G	0.300	0.107	10.001
DXY	DEG/HR/G	0.300	0.069	10.001
DYX	DEG/HR/G	0.300	0.131	10.001
DYY	DEG/HR/G	0.300	0.088	10.001
DZY	DEG/HR/G	0.300	1.602	10.001
DZZ	DEG/HR/G	0.300	6.906	10.001
DXXY	DEG/HR/G2	0.0400	0.1633	1.0001
DYXY	DEG/HR/G2	0.0400	0.2112	1.0001
DZYZ	DEG/HR/G2	0.0400	0.9952	1.0001
GSFX	PPM	300.	1574.	10000.
GSFY	PPM	300.	1328.	10000.
GSFZ	PPM	1000.	9933.	10000.
XGY	ARC MIN	0.667	22.063	30.001
XGZ	ARC MIN	-0.667	3.380	30.001
YGX	ARC MIN	-0.667	22.548	30.001
YGZ	ARC MIN	0.667	6.347	30.001
ZGX	ARC MIN	0.667	21.954	30.001
ZGY	ARC MIN	-0.667	24.440	30.001
ABX	UG	50.	976.	1000.
ABY	UG	50.	967.	1000.
ASFX	PPM	150.	95.	10000.
ASFY	PPM	150.	93.	10000.
YAZ	ARC MIN	-3.001	0.460	30.001

LINEAR COMB.: LIN1=DXF+XGY*WIEZ LIN2=DYF-YGX*WIEZ LIN3=DZF+DZZ*G+GSF*WIEZ

LIN1	DEG/HR	0.00458	0.01468	0.12273
LIN2	DEG/HR	0.00458	0.01856	0.12273
LIN3	DEG/HR	0.00476	0.08197	14.15386

Table 12 Supersonic flight

Source	Units	Sim. Value	Filter Comp Uncertainty	Initial Uncertainty
DXF	DEG/HR	0.00300	0.05721	0.10004
DYF	DEG/HR	0.00300	0.05713	0.10004
DZF	DEG/HR	-0.30321	2.22427	10.00384
DXX	DEG/HR/G	0.300	0.167	10.001
DXY	DEG/HR/G	0.300	0.105	10.001
DYX	DEG/HR/G	0.300	0.170	10.001
DYY	DEG/HR/G	0.300	0.114	10.001
DZY	DEG/HR/G	0.300	0.947	10.001
DZZ	DEG/HR/G	0.300	2.280	10.001
DXXY	DEG/HR/G2	0.0400	0.0539	1.0001
DYXY	DEG/HR/G2	0.0400	0.0624	1.0001
DZYZ	DEG/HR/G2	0.0400	0.9740	1.0001
GSFX	PPM	300.	798.	10000.
GSFY	PPM	300.	1134.	10000.
GSFZ	PPM	1000.	9983.	10000.
XGY	ARC MIN	0.667	24.727	30.001
XGZ	ARC MIN	-0.667	3.387	30.001
YGX	ARC MIN	-0.667	24.772	30.001
YGZ	ARC MIN	0.667	3.587	30.001
ZGX	ARC MIN	0.667	13.048	30.001
ZGY	ARC MIN	-0.667	3.091	30.001
ABX	UG	50.	602.	1000.
ABY	UG	50.	569.	1000.
ABZ	UG	-50.	833.	1000.
ASFX	PPM	150.	37.	10000.
ASFY	PPM	150.	34.	10000.
ASFZ	PPM	150.	858.	10000.
YAZ	ARC MIN	-3.001	0.166	30.001
ZAX	ARC MIN	-0.498	2.050	30.001
ZAY	ARC MIN	0.498	2.208	30.001

LINEAR COMB.: LIN1=DXF+XGY*WIEZ LIN2=DYF-YGX*WIEZ LIN3=DZF+DZZ*G+GSFZ*WIEZ

LIN1	DEG/HR	0.00458	0.01688	0.12273
LIN2	DEG/HR	0.00458	0.01668	0.12273
LIN3	DEG/Hr	0.00476	0.09299	14.15386

and out-and-back flights. The first out-and-back flight is a north-south flight with 84-min legs. The second is an east-west flight with 84-min legs. The third flight is a northeast-southwest flight with 84-min legs, so that the ground track is 45° between the X-Y instrument axes.

The L-shaped flight is clearly superior to the out-and-back flights. Many of the sources of error are determined more accurately in the L-shaped flight than in the better determination of the two cardinal heading out-and-back flights. These sources are the Z gyro Y axis acceleration sensitivity DZy, the two horizontal gyro scale factors $GSFx$ and $GSFy$, the X gyro misalignment about ZXG_z, the horizontal accelerometer scale factor errors $ASFx$ and $ASFy$, and input axis nonorthogonality YAz. Only a few sources of error are determined less accurately by the L-shaped flight, namely four of the gyro input axis misalignments.

The out-and-back flights are not exactly straight line flights. In the north-south flight, the subsonic aircraft turns south by executing a 30° – banked right turn through 225° (180° plus 45°). It flies back to the 106° longitude line on a 45° interception angle. It executes a left turn onto the 106° longitude line and the southerly heading of 180° true. This gives the secondary-axis sources of error an opportunity to contribute to the measureable velocity and position error. And, unfortunately, their contributions apparently are correlated with the contributions of the primary-axis sources of error. The L-shaped flight path is more successful at breaking up these correlations, and, therefore, is more successful at individual error source determination.

In the northeast-southwest interaxis flight, nearly all sources of error are poorly determined because of the strong correlation between the contributions of sources of error from both axes. Only the estimate of the Y accelerometer misalignment about Z has a computed uncertainty smaller than the value of the simulated source of error.

It has thus been shown that the additional maneuvers and flight directions in the L-shaped flight path reduce the correlation of the many sources of error as compared to the simpler out-and-back flights. A more radical departure from simple geometric flight paths is the following: the aircraft holds in the navigate mode for only 5 min, then takes off to the north, climbing for 8 min to 11,000-ft altitude. Then, the aircraft flies 24 different 5-min legs. The heading of each leg was selected by a random number generator (0°-360° uniform probability distribution). The sequence of random headings is the following: 139, 254, 029, 181, 255, 328, 108, 001, 007, 035, 175, 159, 074, 013, 063, 317, 142, 351, 313, 117, 217, 343, 229, 332° true. The aircraft turns to each new heading with a left or right turn, whichever requires the smaller heading change. Bank angles up to 60° are used, depending on the size of the heading change.

The results of the random flight are presented in Table 11. When compared with the L-shaped north-south-east-west flight with 21-min legs (Table 9), the results show a significant improvement in the computed uncertainties of the estimates of: the g-sensitive gyro drifts DXy and DYy, the g^2-sensitive gyro drifts DX_{xy} and DY_{xy}, the accelerometer scale factors $ASFx$ and $ASFy$, and the horizontal accelerometer nonorthogonality YA_z. The results are worse for the horizontal gyro scale factors $GSFx$ and $GSFy$, and for the gyro input axis misalignments XG_z and YG_z. It is clear that more frequent maneuvering in random directions greatly improves the observability of the acceleration sensitive sources of error.

The more intense maneuvers associated with supersonic flight provide greater observability for most of the acceleration-sensitive sources of error. This is demonstrated in the following example.

The aircraft holds on the runway for 5 min, then takes off to the north and executes a minimum-time climb to 65,000-ft altitude. The climb includes an acceleration at low altitude to almost Mach 1, a climb to 35,000 ft below Mach 1, a dive through the transonic drag rise, a supersonic climb to Mach 1.75 and 35,000 ft and a zoom to final altitude. After reaching 65,000 ft and while maintaining a speed near Mach 1 the aircraft flies 15 random headings, selecting a new heading every one minute. The sequence of headings includes the first 15 headings of the subsonic random flight presented above. The short flight duration is because of the limited endurance of supersonic aircraft. A high-accuracy reference system is assumed, providing both horizontal and vertical position and velocity information. One measurement set per minute is utilized. The vertical channel state variables are included.

The results are presented in Table 12. This flight provides the first reasonably satisfactory estimates of two of the g^2–sensitive gyro drift coefficients $DXxy$ and $DYxy$. The estimates of horizontal accelerometer scale factor errors ASF_x and ASF_y and input axis nonorthogonality YA_z are excellent—the best of any flight attempted regardless of duration. The determination of the g-sensitive gyro drift coefficients is comparable to the accuracy achieved in several of the longer subsonic flights. Some of the angular velocity sensitive errors are determined as well as in the 2-hr north-south-

Table 13 Strapdown flight test

Source	Units	Sim. Value	Filter Comp Uncertainty	Initial Uncertainty
DXF	DEG/HR	0.02500	0.99602	1.00038
DYF	DEG/HR	0.01800	0.03811	1.00038
DZF	DEG/HR	0.01800	0.01845	1.00038
DXX	DEG/HR/G	0.200	1.003	10.001
DXZ	DEG/HR/G	0.200	9.402	10.001
DYY	DEG/HR/G	0.200	1.036	10.001
DYZ	DEG/HR/G	0.200	7.018	10.001
DZZ	DEG/HR/G	0.200	4.308	10.001
DZY	DEG/HR/G	0.200	1.478	10.001
DXXZ	DEG/HR/G2	0.0700	1.0000	1.0001
DYXZ	DEG/HR/G2	0.0700	0.9976	1.0001
DZXY	DEG/HR/G2	0.0700	0.9952	1.0001
GSFX+	PPM	70.	356.	10000.
GSFX-	PPM	70.	416.	10000.
GSFY+	PPM	70.	331.	10000.
GSFY-	PPM	70.	309.	10000.
GSFZ+	PPM	70.	298.	10000.
GSFZ-	PPM	70.	943.	10000.
XGY	ARC MIN	0.167	2.105	30.001
XGZ	ARC MIN	-0.167	1.321	30.001
YGX	ARC MIN	-0.167	1.480	30.001
YGZ	ARC MIN	0.167	2.041	30.001
ZGX	ARC MIN	0.167	1.471	30.001
ZGY	ARC MIN	-0.167	0.718	30.001
ABX	UG	30.	971.	1000.
ABY	UG	20.	535.	1000.
ABZ	UG	20.	202.	1000.
ASFX	PPM	35.	991.	10000.
ASFY	PPM	35.	1232.	10000.
ASFZ	PPM	35.	4418.	10000.
XAY	ARC MIN	-0.167	23.517	30.001
XAZ	ARC MIN	0.167	14.439	30.001
ZAX	ARC MIN	-0.167	3.510	30.001

east-west flight with 21-min legs. These include the horizontal gyro scale factor errors GSF_x and GSF_y and the gyro input axis misalignments XG_z and YG_z. The major changes in vertical velocity provide some observability of the accelerometer scale factor error ASF_z and its misalignments ZA_x and ZA_y.

It is interesting to note that accelerometer biases are theoretically observable, as indicated by the reduction of uncertainty from initial to final value. If the aircraft follows a free-fall trajectory (zero specific force) for a significant period of time, the observed specific force measurement error must be due to accelerometer bias or gravity model error. The supersonic flight has such a period during the transition from subsonic climb to the dive through drag rise. The improved accelerometer bias estimates translate into better platform attitude error estimates, which, in turn, permit better estimates of the Z accelerometer input axis misalignments ZA_x and ZA_y.

Flight Test of Strapdown INS.

Laboratory testing was shown to be very effective for the determination of the individual error coefficients in a strapdown system error model. Flight testing, on the other hand, does not provide such strong observability, as is shown by the following example. Nevertheless, flight testing is necessary because the assumed error model may not contain all the significant error modes.

The simulated strapdown INS is installed in the aircraft with its X axis down, the Y axis toward the aircraft nose, and the Z axis toward the right wing. The aircraft flies the 2-hr random heading subsonic flight. The sequence of 24 random headings is the same as in the test of the local level INS. A difference is that 30° bank angles (rather than 60° bank angles) are used for the turns in this strapdown test. The high-accuracy reference measurements are assumed available in all three axes. The measurement rate used by the post-test filter is one set per 3 min.

The filtering results are presented in Table 13. The results are quite diasppointing in that no source of error is estimated with a computed uncertainty smaller than the simulated value. Only the lateral gyro g-insensitive drift rate DZ_f has a computed uncertainty comparable to the simulated value.

The reason that most of the g-sensitive sources of error are poorly observed is that, with coordinated turns, there is little or no lateral specific force f_z, and with moderate speed changes, climbs, and descents the longitudinal specific force f_y is small. Hence, all the g-sensitive gyro drifts that are related to f_z or f_y are poorly observed. The g^2-sensitive gyro drifts are all related to f_z and/or f_y and are not observed. The lateral and longitudinal accelerometer scale factor errors ASF_z and ASF_y are poorly observed. The normal accelerometer misalignments XA_y and XA_z are poorly observed.

It is interesting to note that the lateral and longitudinal accelerometer biases AB_z and AB_y are more observable in a strapdown flight test than are the horizontal accelerometer biases in a local-level gimbaled system flight test. The reorientation of the strapdown accelerometers for each new heading breaks up the correlation with the system attitude errors.

Reference System Effect on Observability

The effect of some of the reference system characteristics on the observability of error sources is explored in this section. The varied parameters consist of the accuracy of the reference system, its measurement rate, and the accuracy of the knowledge of gravity.

The north-south-east-west flight with 21-min legs was used for these tests. In the first test, a reference system with an accuracy of 200-ft one sigma in east and north position, and no velocity information, was assumed. This is the accuracy obtainable, for example, by photographing surveyed checkpoints using an aircraft-mounted vertically stabilized camera. The second test is with the baseline high-accuracy reference system, but the reference system measurement rate is increased from one set every 3 min to one set per minute. The third test assumed that an accurate gravity survey was available and used to compensate the INS data, and that the remaining residual errors in the deflection survey data were only 10% of the original values.

The results are presented in Table 14. Also included are the results obtained with the same flight in the nominal configuration.

For the degraded reference system case, the most significant accuracy differences are in the estimates of the accelerometer scale factor errors ASF_x and ASF_y and the horizontal accelerometer input axis nonorthogonality YA_z. The high-accuracy reference provides a factor of three improvement in these estimates. A factor of three improvement is also obtained for the estimate of the g-sensitive gyro drift coefficient DZ_y. Factor of two improvements are obtained for the gyro scale factor errors GSF_x and GSF_y and for the gyro input axis misalignment XG_z. All other sources of error receive less than a factor of two benefit.

For the increased measurement rate case, there is no dramatic improvement in the determination of any source of error, such as might be due to catching some critical information just after a maneuver. The imporved performance in no case is better than the factor of $\sqrt{3} = 1.73$ to be expected from measurement averaging effects. The best improvements are in the estimates of accelerometer scale factor errors ASF_x and ASF_y and the input axis nonorthogonality YA_z. Lesser improvements are seen in the g-sensitive gyro drift coefficient DZ_y, the gyro input axis misalignments ZG_x and ZG_y, and the azimuth angular velocity error linear combination $LIN3$.

In the case where an accurate gravity survey is available, the best accuracy improvements seen are the factor of two improvements in the estimates of a gyro scale factor GSF_y and a gyro input axis misalignment YG_z. All other estimates receive a smaller factor improvement. The gravity deflection is a significant error source, but does not seem to be the dominant factor preventing accurate determination of all sources of error.

Conclusions

The major problem preventing more accurate determination of the dozens of sources of error in an inertial navigation system is the high correlation between the contributions of many of the sources of error. Different lab and

Table 14 Reference system effects on observability

Source	Units	Sim. Value	Reduced Reference Accuracy Filter Comp Uncer	1 set per 1 min Filter Comp Uncer	0.1 Nominal Deflections Filter Comp Uncer	Baseline Filter Comp Uncer
DXF	DEG/HR	0.00300	0.05642	0.05331	0.05279	0.05465
DYF	DEG/HR	0.00300	0.05618	0.05475	0.05112	0.05524
DZF	DEG/HR	-0.30321	7.00544	6.76619	6.73109	6.82405
DXX	DEG/HR/G	0.300	0.183	0.108	0.089	0.117
DXY	DEG/HR/G	0.300	0.215	0.125	0.108	0.133
DYX	DEG/HR/G	0.300	0.180	0.133	0.086	0.135
DYY	DEG/HR/G	0.300	0.211	0.151	0.108	0.152
DZY	DEG/HR/G	0.300	6.507	1.644	1.709	2.087
DZZ	DEG/HR/G	0.300	7.003	6.743	6.709	6.807
DXXY	DEG/HR/G2	0.0400	0.9877	0.8076	0.8052	0.8644
DYXY	DEG/HR/G2	0.0400	0.9876	0.8539	0.8145	0.8880
DZYZ	DEG/HR/G2	0.0400	0.9972	0.9952	0.9953	0.9953
GSFX	PPM	300.	2225.	794.	639.	880.
GSFY	PPM	300.	1326.	741.	405.	735.
GSFZ	PPM	1000.	9997.	9986.	9986.	9992.
XGY	ARC MIN	0.667	24.604	22.572	22.115	23.036
XGZ	ARC MIN	-0.667	4.575	1.907	1.424	2.058
YGX	ARC MIN	-0.667	24.622	23.278	21.653	23.467
YGZ	ARC MIN	0.667	4.878	3.277	1.770	3.261
ZGX	ARC MIN	0.667	27.780	16.205	17.030	19.686
ZGY	ARC MIN	-0.667	29.092	16.395	18.244	19.864
ABX	UG	50.	989.	901.	889.	932.
ABY	UG	50.	989.	874.	883.	919.
ASFX	PPM	150.	596.	119.	135.	166.
ASFY	PPM	150.	623.	120.	161.	192.
YAZ	ARC MIN	-3.001	2.526	0.578	0.767	0.899
LINEAR COMB.:		LIN1=DXF+XGY*WIEZ		LIN2=DYF-YGX*WIEZ	LIN3=DZF+DZZ*G+GSFZ*WIEZ	
LIN1	DEG/HR	0.00458	0.01851	0.01024	0.00921	0.01083
LIN2	DEG/HR	0.00458	0.01756	0.01169	0.00858	0.01173
LIN3	DEG/HR	0.00476	0.10427	0.06095	0.06325	0.07611

flight tests have been investigated to identify those tests that provide the greatest enhancement in the observability of error sources.

The laboratory tests considered were limited to the testing of complete systems in the navigate mode. No precision attitude references were used. Outputs from the INS under test included the indicated position and velocity. The system is operated in the navigate mode. Special test modes were not considered.

Under these ground rules, laboratory tests of local-level systems provide very little information about the INS sources of error. Only the two horizontal gyro scale factor errors GSF_x and GSF_y were determined to an accuracy (computed uncertainty) better than the value of the error source in the simulated local-level accurate INS. To accomplish the determination of these gyro scale factor errors required a four-alignment test procedure. This procedure is possible with the wander azimuth INS, but may not be possible without special test modes with other local-level INS.

Laboratory testing of strapdown systems is much more productive. A laboratory test sequence was shown which made almost all sources of error observable. Only the three accelerometer scale-factor-error estimates had computed uncertainties larger than the values of the simulated error sources. The demonstrated test sequence requires a rotating inertial test table with a two-degree-of-freedom mount. Precision attitude is not required. The table and mount angles need not be recorded if the inertial-indicated attitude can be obtained, along with the indicated velocity and position.

Many different flight paths have been simulated to discover better flight paths for the determination of sources of error. Most tests were conducted with the local-level system. A set of north-south-east-west L-shaped flight paths was simulated, the leg durations varying from 12 min to 84 min. The acceleration-sensitive sources of error are determined more accurately with the 21-min legs than with the 12-min legs, but increasing the leg durations to 42 or 84 min does not yield a further improvement. This shows that long-distance flights are not required for determination of the acceleration-sensitive error sources. These error sources are the g-sensitive and g^2-sensitive gyro drift coefficients, the accelerometer scale factor errors, and the accelerometer input axis misalignments. The error sources that *are* determined more accurately with longer flights are the g-insensitive gyro drifts, the gyro torquer scale factor errors, and the gyro input axis misalignments.

The flight test with 42-min legs produced results better than the test with 21-min legs but not as good as the test with 84-min legs. Nothing particularly good or bad was noted in the flight with 42-min legs.

A north-south and an east-west out-and-back flight, each along a horizontal axis of the local level INS, were compared with an L-shaped flight of the same duration. The L-shaped flight path was clearly superior in that it determined most sources of error as accurately or more accurately than the better determination of the two out-and-back flights.

An out-and-back flight on a heading 45° between the horizontal axis of the local level INS failed to determine accurately all sources of error except the horizontal accelerometer input axis nonorthogonality. The problem of error source correlation is particularly severe for this interaxis out-and-back flight.

It is clear that introducing more maneuvers improves the observability of the acceleration sensitive errors. A 2-hr flight with 24 5-min legs, whose headings were selected by a random number generator, produced significantly better results for these errors.

A supersonic flight having a minimum-time climb plus a 15-min period of random-heading supersonic flight (15 headings of 1 min duration each) provided the best determination of the gyro g^2-sensitive drift coefficients, the horizontal accelerometer scale factor errors, and the horizontal accelerometer input axis nonorthogonality. The determination of the g-sensitive gyro drift coefficients is comparable to the accuracy achieved in several of the longer subsonic flights. Some of the angular velocity sensitive errors are determined as well as in a 2-hr subsonic flight, but not as well as in longer subsonic flights.

A flight test of the strapdown system was shown to be nowhere near as effective as the laboratory test in determining the individual coefficients in the assumed INS error model. One problem is that the specific force in coordinated aircraft turns remains parallel to the normal axis and, hence, does not change direction with respect to the strapdown gyros and accelerometers. This prevents the acceleration-sensitive sources of error from being separately excited. Nevertheless, flight testing of strapdown systems is necessary because the assumed error model may not contain all the significant error modes.

The effect of reference system accuracy on INS source-of-error determination accuracy was explored. A surprising result was that, over the same flight paths, the increased accuracy of the reference system does not improve the determination of INS error sources by more than a factor of three. Most sources of error receive a lesser factor of improvement.

A faster measurement rate of one set per minute compared with one set per 3 min yielded improved source-of-error accuracy. But in no case was the improvement better than the factor of square root of three associated with averaging the greater number of measurements. Whether or not such accuracy improvement actually can be obtained depends on the correlation time associated with the reference system errors.

If an accurate survey of the gravity deflection is available along a selected flight path, by using this information to eliminate this source of error from the INS data, the accuracy of the estimates of a few of the gyro sources of error can be improved by about a factor of two. All other estimates are improved by smaller amounts.

There was found no one best flight path for the testing of inertial navigation systems. Among the subsonic flight paths, the 2-hr flight with the many different (randomly selected) headings provided the best observability for the acceleration-sensitive sources of error. A short supersonic flight test provided even better observability of the acceleration-sensitive sources of error. But the best flights for observing the constant gyro drift rates and the rate-sensitive gyro errors were the longer subsonic flights, such as the north-south-east-west flight with 84-min legs. Given a subsonic aircraft with sufficient endurance,an excellent flight path would be the combination of the north-south-east-west flight with 84-min legs plus the 2 hr of relatively local flying with frequent random heading changes.

Appendix

The conventional discrete version of the Kalman filter equations propagates the error covariance matrix P between time instants t_k, t_{k+1} according to

$$P_{k+1} = \phi_k P_k \phi_k{}^T + Q_k$$

where P_{k+1} is the error covariance matrix at time t_{k+1}. The state transition matrix ϕ is a time-varying matrix, whose elements are functions of the test sequence velocity or acceleration profile or functions of INS attitude.[9] The process noise matrix Q is used to model the random fluctuations in some of the filter states, as described by the error source statistics in Tables 3 and 5.

At time t_k, a measurement set is optimally combined with the current state. The error covariance matrix P_k is updated by the measurement set according to

$$P_k{}^+ = P_k - K_k H_k P_k \qquad K_k = P_k H_k{}^T (H_k P_k H_k{}^T + R_k)^{-1}$$

Where $P_k{}^+$ is the updated error covariance matrix following the measurement incorporation. R_k is the matrix of measurement noise covariance, and is specified by the measurement random error statistics detailed in Table 1. The measurement gradient matrix H_k relates the filter states to the measurements. As seen from Table 1, the assumed measurement set is directly related to the test INS position and velocity errors; therefore, H_k contains only zeros or ones in appropriate locations.

The assumed initial covariance matrix is diagonal, with large values, relative to the simulated error sources, assigned for each of the filter states. In this fashion the filter estimates are a function of the flight test measurements and not a function of a priori estimates. The square-root of these diagonal values for the modeled INS error sources are included in some of the tables in the initial uncertainty column.

The simulation technique used to determine INS error source observability is to exercise the Kalman estimator over the test profile, and examine the filter final covariance matrix. The square root of the final covariance matrix diagonal values represent the confidence of the filter in its estimates, and are included in the tables, for the INS source-of-error states, in the final uncertainty column.

References

[1] Downs, H.B., "Notes on Flight Paths," WP-GTA-70-3, Central Inertial Guidance Test Facility, Holloman Air Force Base, N.M., Sept. 1970.

[2] Kalman, R.E., "A New Approach to Linear Filtering and Prediction Problems," *Transactions of the ASME-Journal of Basic Engineering*, Ser. D., Vol. 82, March 1960, pp. 35-45.

[3] Kaminski, P.G., Bryson, A.E., Jr., and Schmidt, S.F., "Discrete Square Root Filtering: A Survey of Current Techniques," *IEEE Transactions on Automatic Control*, Vol. AC-16, Dec. 1971, pp. 727-736.

[4] Carlson, N.A., "Fast Triangular Formulation of the Square Root Filter," *AIAA Journal*, Vol. 11, Sept. 1973, pp. 1259-1265.

[5] Britting, K.R., *Inertial Navigation System Analysis*, Wiley-Interscience, New York, 1971.

[6] Widnall, W.S. and Grundy, P.A., "Inertial Navigation System Error Models," TR-03-73 (DDC AD 912489), Intermetrics, Inc., Cambridge, Mass., May 1973.

[7] Nash, R.A., Kasper, J.F., Jr., Crawford, B.S., and Levine, S.A., "Application of Optimal Smoothing to the Testing and Evaluation of Inertial Navigation Systems and Components," *IEEE Transactions on Automatic Control*, Vol. AC-16, Dec. 1971, pp. 806-816.

[8] Fraser, D.C., "A New Technique for the Optimal Smoothing of Data," T-474, MIT Instrumentation Lab., Cambridge, Mass., Jan. 1967.

[9] Widnall, W.S. and Grundy, P.A., "Inertial Navigation System Test Methods," TR-08-73, Intermetrics, Inc., Cambridge, Mass., Aug. 1973.

[10] Birnbaum, M., "Aircraft Navigation Systems Testing with a High-Precision Reference," AGARD Lecture Ser. 60 on Testing Philosophy and Methods of Guidance and Control Systems and Subsystems, NITS, Springfield, Va., Oct. 1973.

[11] Widnall, W.S., "Filtering and Smoothing Simulation Results for CIRIS Inertial and Precision Ranging Data", *AIAA Journal*, Vol. 12, June 1974, pp. 856-861.